Order analysis method based on instantaneous phase

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Abstract

Order analysis is an effective method to analyse non-stationary signal of rotating machinery. The key of this method is to acquire the time sequence under even angle re-sampling. This paper proposed an order analysis method based on instantaneous phase using Hilbert-Huang Transform (HHT), and achieved the order spectrums of torsional vibration signals of rotating machinery from simulation and experiment. Being different from the order analysis method based on instantaneous frequency, this method directly uses the instantaneous phases obtained by HHT to get rotating angle over time. Thus, it is faster and more convenient. Meanwhile, this method is less affected by the 'boundary effect'. Hence, it can achieve higher precision. The simulating and experimental analysis both verified the feasibility and accuracy of this method.

Keywords: order analysis, even angle sampling, HHT, instantaneous phase

1 Introduction

Order analysis method is a highly effective approach to analyse the non-stationary vibration signals of rotating machinery [1]. To achieve this method, the key is to acquire the time sequence of order sampling according to the base speed or frequency of the rotating shaft. The traditional way is to install a phase pulse generator on the shaft system [2-3], which increases the complexity of the shafting and also generates signal synchronization problems. Moreover, sometimes it is not convenient to install the other sensors on the rotating shafts. As a result, scholars at home and abroad have put forward many order analysis methods without tachometers. Shao. H et al put forward an order tracking algorithm based on instantaneous frequency estimation using Gabor transform [4]; Guo Yu et al fulfilled an instantaneous frequency estimation order tracking method based on Short-time Fourier transform spectrum [5]; Xu Minqiang et al achieved an instantaneous frequency estimation method of rotating machinery vibration signals in high speed start process for Haar wavelet analysis [6]; Jia Jide et al realized an order analysis method based on instantaneous frequency estimation using Hilbert-Huang (HHT) transform [7]. Since HHT is not influenced by Heisenberg uncertainty principle, it is more effective to achieve can achieve multi-component signals analysis than the others. Therefore, it becomes a new approach in order analysis with its unique advantages.

The implementation steps of the order analysis method based on instantaneous frequency estimation using HHT are as follows [7]:

Firstly, complex signal was divided into a finite number of Intrinsic Mode Functions (IMFs) by Empirical Mode Decomposition (EMD). Secondly, for main components of the signal, the Hilbert Transform was used, obtaining the instantaneous frequency. Thirdly, the rotating speed was estimated by the instantaneous frequency, and then the rotating angle over time was decided by the speed. Finally, adopting even angle re-sampling on original signal, the angle domain signal was gotten, after conducting spectral analysis, the order spectrum of the original signal was acquired. The instantaneous frequency computed by HHT was influenced by 'boundary effect' and 'Gibbs effect', which introduced serious errors to the order analysis.

Aiming at the error issue brought by the instantaneous frequency acquired by HHT, this paper proposed an order analysis method based on instantaneous phase. Using the instantaneous phase information of the main component acquired by HHT directly, the corresponding relation between the rotating angle and time was plotted. Then according to the rotating angle, the time sequence of even angle re-sampling was obtained, and finally the order analysis was achieved. The simulating and experimental analysis both verify the feasibility of this method. It is simple and reliable, which provides a new approach to analyse the non-stationary signal of rotating machinery.

2 Theories of order analysis method based on instantaneous phase

2.1 EMPIRICAL MODE DECOMPOSITION

HHT is based on the study of instantaneous frequency of non-stationary signal. Usually, we define a signal's instantaneous frequency as the differential of its instantaneous phase. With regard to a multi-component

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signal, there will be several instantaneous frequencies at a certain time. In order to make the definition of the instantaneous frequency owing a physical meaning, Norden E Huang proposed to decompose the signal into mono-component, which was IMF. Each IMF should meet the following two demands:

1) in the whole data set, the number of zero crossings and the number of extrema must either equal or differ at most by one;

2) at any point, the mean value of the envelopes decided by the local maxima and local minimum point is zero, namely, the signal is axisymmetric by time axis [8]. For signal $x(t)$, the EMD algorithm is as follows [9]:

Step 1 Determine all extrema of the signal (maximas and minimas). Then interpolate them and fit the points by cubic spline curve, obtaining the upper envelopes $e_u(t)$ and (lower envelopes) $e_l(t)$;

Step 2 Calculate the mean value of the upper and lower envelope, getting $m_1(t)=(e_u(t)+e_l(t))/2$, and then make the original signal sequence minus the mean value, acquiring $h_1(t)=x(t)-m(t)$;

Step 3 Repeat the above steps, until $h_k(t)$ meets the IMF requirements: $c_1(t)=h_k(t)$;

Step 4 Calculate the difference between $x(t)$ and $c_1(t)$, yeilding the residual $r_1(t)=x(t)-c_1(t)$, and iterate all above steps until the final residual $r_n(t)$ becomes a monotonic function.

After EMD, the original signal $x(t)$ is composed of n IMFs and the final residual component, as shown in Equation (1). The residual component is a mean trend or a constant:

$$x(t)=\sum_{i=1}^{n}c_i(t)+r_n(t). \tag{1}$$

2.2 HILBERT TRANSFORM

The Hilbert Transform is applied to each IMF component obtained from EMD:

$$H(c_i(t))=\frac{1}{\pi}P\int_{-\infty}^{\infty}\frac{c_i(t')}{t-t'}dt', \tag{2}$$

where *P* is the Cauchy principal value of this integral. $c_i(t)$ and $H(c_i(t))$ form a complex conjugate pair, then an analytic signal $z_i(t)$ can be gotten:

$$z_i(t)=c_i(t)+iH(c_i(t))=a_i(t)e^{i\varphi_i(t)}. \tag{3}$$

Then, the instantaneous amplitude is shown as:

$$a_i(t)=[c_i(t)+H(c_i(t))]^{1/2}. \tag{4}$$

The instantaneous phase can be obtained from Equation (5):

$$\varphi_i(t)=\arctan\frac{H(c_i(t))}{c_i(t)}. \tag{5}$$

The instantaneous frequency is shown as Equation (6):

$$f_i(t)=\frac{1}{2\pi}\frac{d\varphi_i(t)}{dt}. \tag{6}$$

2.3 INSTANTANEOUS PHASE PROCESSING

With regard to the rotating shafts of a rotating machinery, the phase of the base frequency component is correspond to the rotating angle of the shafts, and the phases of the multiple frequency components are integer multiples of rotating angle of the shafts. The instantaneous phases acquired by HHT always keeps $\varphi_i(t)\in(-\pi,\pi)$, which are not continuous. In order to obtain continuous phase information, which is the real rotating angle, the instantaneous phases should be processed.

The steps of the instantaneous phase processing are as follows:

1) Make the first point to be zero, which means that the first point of the curve should be processed to be the component's original phase.

2) Deal with the breakpoint. When the curve appears mutation from π to $-\pi$, connect each breakpoint and add 2π to the connection point.

3) After acquiring the continuous phase, use the quadratic polynomial to fit it.

Then, the instantaneous phase will be changed as follows:

$$\varphi_i'(t)=b_0+b_1t+b_2t^2. \tag{7}$$

2.4 EVEN ANGLE RE-SAMPLING

The phase after curve fitting reflects the relation between the rotating angle and time. In order to obtain the angle-domain signal, each time increment related to the even angles should be calculated. Assuming the even angle increment is $\Delta\theta$, the number of sampling points in angular domain (FFT points) is *N*, the corresponding time sequence is $\{T_j\}$, $j=1,2,\cdots N$. Therefore, the followed equation can be obtained:

$$\begin{cases} b_0+b_1T_1+b_2T_1^2=0 \\ b_0+b_1T_2+b_2T_2^2=\Delta\theta \\ \cdots \\ b_0+b_1T_N+b_2T_N^2=(N-1)\cdot\Delta\theta \end{cases}, \tag{8}$$

where $\Delta\theta$ is determined by the maximum ratio range order:

$$\Delta\theta=2\pi/\mathrm{O}_{\max}, \tag{9}$$

where $\mathrm{O}_{\max}$ is the maximum ratio range order. Thus, according to Equation (8), the time sequence known as $\{T_j\}$ can be calculated.

The time sequence of the original time-domain signal $x(t)$ in time domain is $\{t_m\}$, where $m = 1,2,\ldots,m,\ldots,M$ and M is the sampling points.

Appling the linear interpolation to the original signal $x(t)$ in the light of the time sequence $\{T_j\}$, the angle domain signal can be obtained, shown in Equation (10).

$$x(T_j) = x(t_m) + \frac{x(t_{m+1}) - x(t_m)}{t_{m+1} - t_m} \cdot (T_j - t_m), \tag{10}$$

where $t_m \le T_j \le t_{m+1}$.

3 Simulations

This paper took the torsional vibration of the rotating machinery as an example, which usually was a multi-components signal with a base frequency corresponded to the rotating speed, described in Equation (11):

$$x(t) = \sum_{i=1}^{n} A_i \cos(2\pi i f_0 t + \varphi_i), \tag{11}$$

where f_0 is the frequency corresponded to the rotating speed. A compound signal being composed of three chirp signals was established, shown in Equation (12). The signal in time-domain was plotted, illustrated in Figure 1. It was assumed that the signal didn't include noise.

$$S = S_1 + S_2 + S_3 = \\ 0.5\sin(2\pi f_0 t) + \sin(2\pi 2 f_0 t) + 1.5\sin(2\pi 4 f_0 t). \tag{12}$$

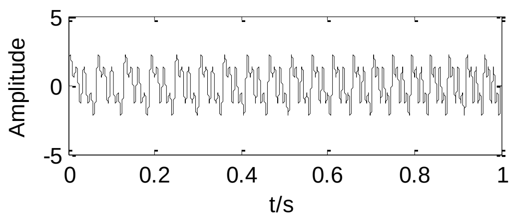


FIGURE 1 Simulating signal

If the rotating speed accelerated from 900r/min to 1200r/min within one second, the base frequency increased from 15Hz to 20Hz, we have:

$$f_0 = 15 + 5t. \tag{13}$$

Applying EMD, the original signal was decomposed into 6 IMFs and one residual component, shown in Figure 2. IMF1, IMF2 and IMF3 were corresponding to the real signal. IMF1 was equal to the quadruple frequency component; IMF2 and IMF3 were corresponded to the double and base frequency components, respectively. However, IMF4, IMF5 and IMF6 were not equal to the real signal because of over decomposition, which was caused by the use of cubic spline curve to form envelopes and the small threshold value of the 'screening'.

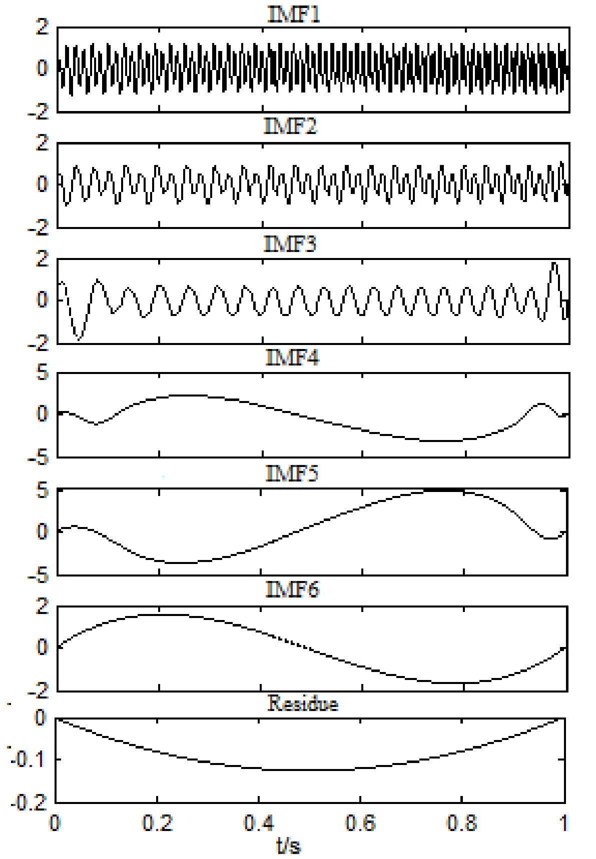


FIGURE 2 EMD of original signal

Appling Hilbert transform to IMF1, IMF2 and IMF3, the instantaneous frequency and instantaneous phase curves were acquired, described in Figure 3 and Figure 4.

The Hilbert spectrum clearly reflected the trend of the instantaneous frequencies of IMF1, IMF2 and IMF3. As shown in Figure 3, however, 'boundary effects' occurred obviously at the beginning and the end of the curve. What's more, the instantaneous frequencies fluctuated visibly because of 'Gibbs effect'. 'Boundary effects' and 'Gibbs effect' will increase errors of the following analysis [7].

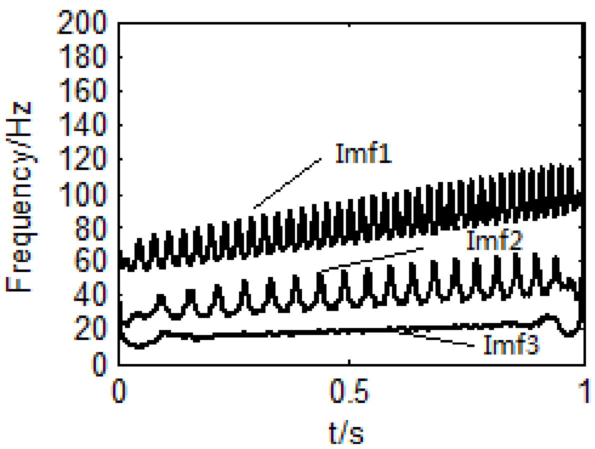


FIGURE 3 Hilbert spectrum

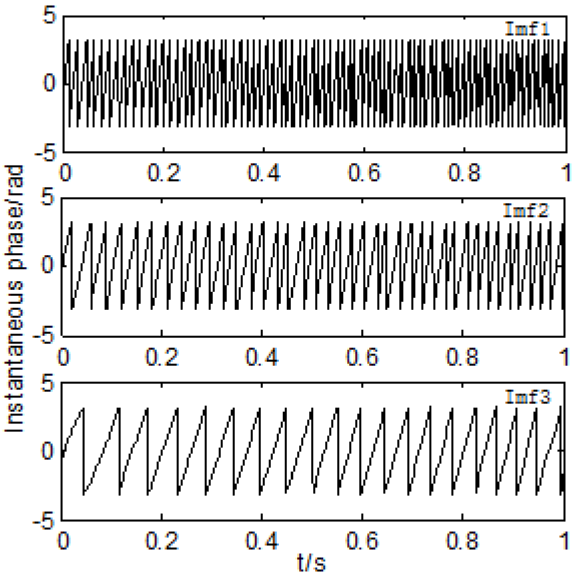


FIGURE 4 Instantaneous phase in time

After processing the instantaneous phases, the continuous phases of IMF1, IMF2 and IMF3 were obtained firstly, shown in Figure 5.

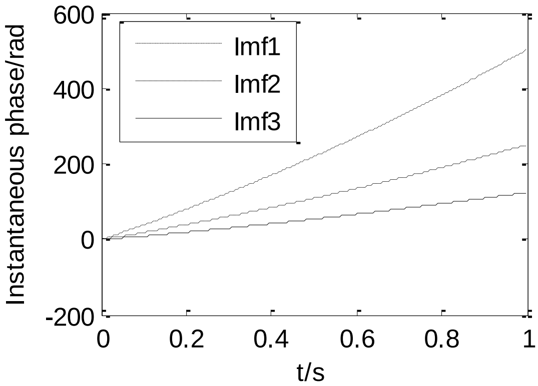


FIGURE 5 Continuous phase in time

$\phi_i(t)$ indicated the continuous phases, where $i = 1,2,3$, therefore:

$$\phi_1(t) = 2\phi_2(t) = 4\phi_1(t). \tag{14}$$

Extracting the instantaneous phases of the base frequency component or the multiple frequency components, the rotating angle changed by time can be obtained. Figure 6 showed the continuous phases of IMF1, IMF2 and IMF3 after quadratic polynomial curve fitting and the actual phase curve of the simulating signal, where the actual phase is shown in Equation (15).

$$\theta(t) = 2\pi(10 + 5t)t\ , \tag{15}$$

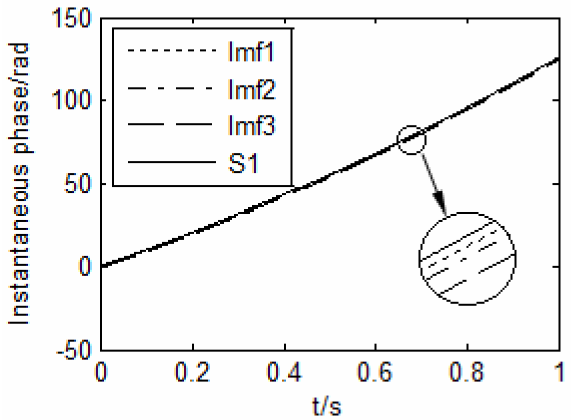
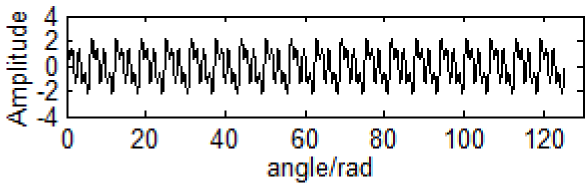
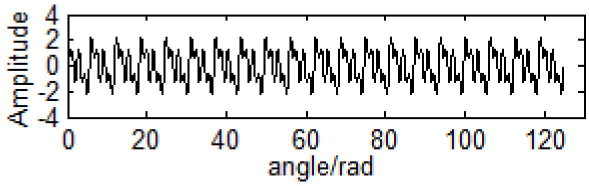


FIGURE 6 Fitting continuous phase in time

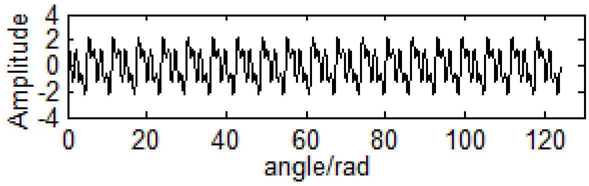
Figure 6 showed that the phases obtained by this method were close to the real phase of the original signal, and the error is small. According to the continuous phases, the angle domain signals were acquired through the re-sampling and interpolation of the original signal, as shown in Figure 7.



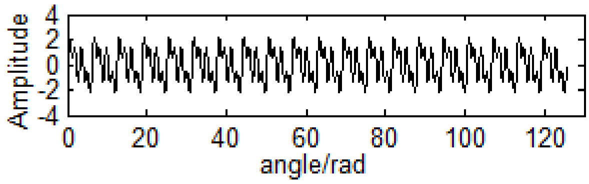
a) Resample signal in angle domain phase 1



b) Resample signal in angle domain using phase 2



c) Resample signal in angle domain using phase 3



d) Resample signal in angle domain using theta

FIGURE 7 Resample signal in angle domain

Appling FFT operation to the angle domain signals, the order spectrums of original signal were obtained, illustrated in Figure 8. It was shown that the order spectrums and the one analysed by actual phase coincided. The results show that the order spectrum can be yielded through the phase information of each component of the original signal.

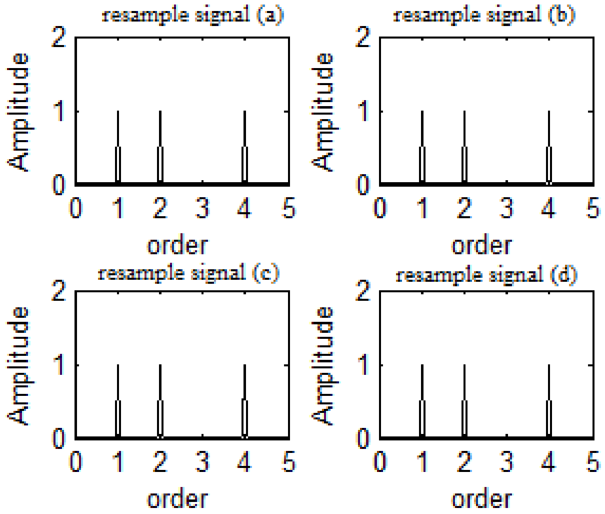


FIGURE 8 Order spectrum

The above results were obtained when the original signal was without noise. Since EMD is not capable to eliminate noise automatically, Hilbert spectrum will occurs large deviation at noisy conditions. Figure 9 showed the order spectrum of the original signal with noise by the illustrated method, the SNR of which is 5. It was shown that apparent frequency confusion and amplitude error existed. Consequently, signal denoising is significant before EMD.

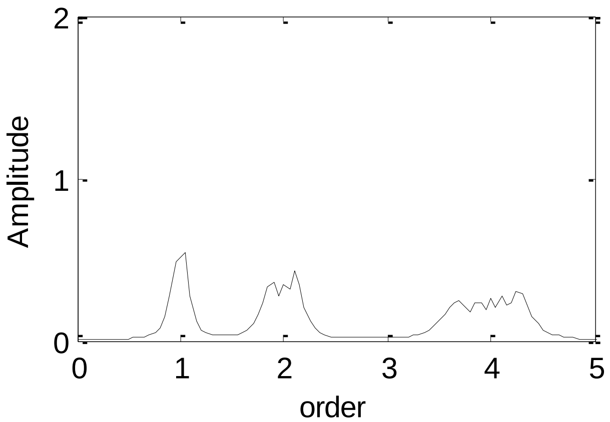


FIGURE 9 Order spectrum (SNR=5)

4 Implementation of the method

In summary, the implementation process of the order analysis based on instantaneous phase is as follows:

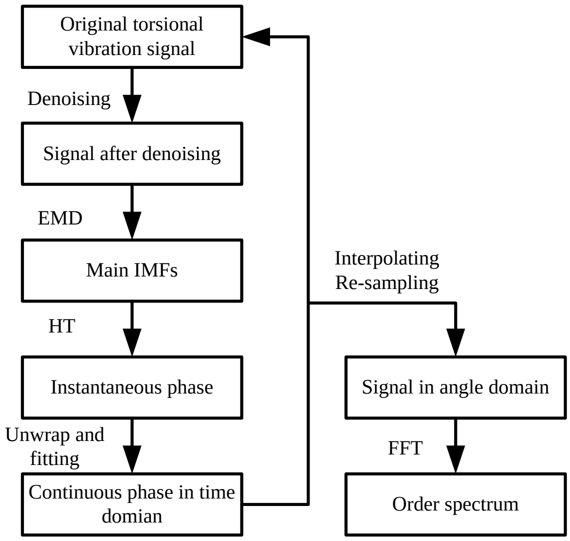
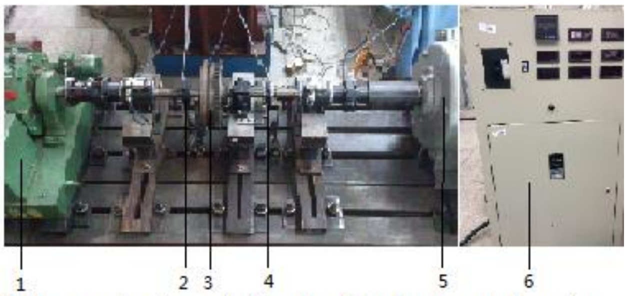


FIGURE 10 Implementation process of the method

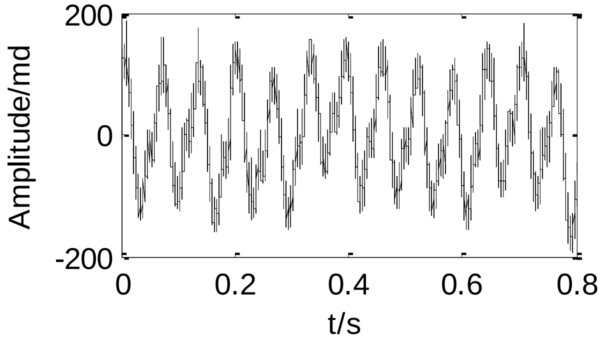
5 Experiments and verification

Torsional vibration test was implemented on an dynamic characteristics test bench of a vehicle torsional damper. Figure 11 showed the test bench. The dedicated frequency inverter was used to control the motor, which generated torsional vibration. The eddy current dynamometer simulated load, and the encoders were used to detect the rotating speed of the shaft system. The test was carried out under a constant load condition. The motor speed controlled by the inverter increased from 900r/min to 1200r/min within 5s, where the angular acceleration was π rad/s2. Since the encoder generated 1440 pulses when the shaft rotated one circle, the sampling frequency was set to be 100 KHz. The data within 0.8s was intercepted to be processed by pulse interval method [11], yielding the torsional vibration data, as shown in Figure 12a. After wavelet denoising [12-13], the torsional vibration data was shown in Figure 12b.

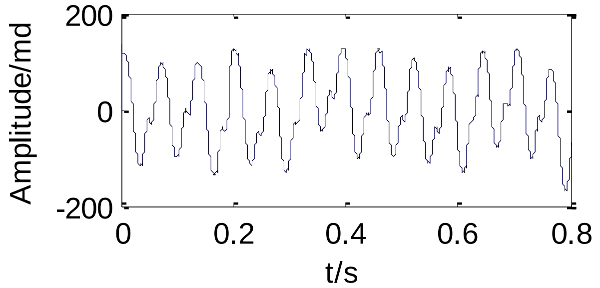


1.dynamometer ; 2.encoder; 3.torsional vibration damper; 4.encoder;
5. converter motor; 6. transducers

FIGURE 11 Torsional vibration test bench



a) Original torsional vibration signal



b) torsional vibration signal after denoising

FIGURE 12 Torsional vibration 3signal

Using EMD, the torsional vibration signal was decomposed, obtaining the IMFs, where IMF1 and IMF2 corresponded to the original signal. Then by Hilbert transform of IMF1and IMF2, the Hilbert spectrum of the signal can be obtained, illustrated in Figure 13. It showed that IMF2 was corresponding to the base frequency component of the torsional vibration signal. Thus, order analysis of the original signal was based on the instantaneous phase of IMF2. Figure 14 described the instantaneous phase of IMF2, and Figure 15 showed the continuous phase curve over time. According to the continuous phase curve, the interpolation re-sampling was implemented, getting the signal in angular domain, shown in Figure 16. Finally, the angular domain signal was calculated by FFT, which obtained the order spectrum of the original signal, as shown in Figure 17.

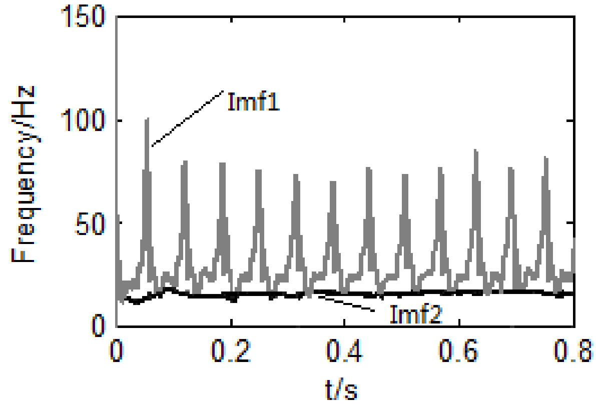


FIGURE 13 Hilbert Spectrum

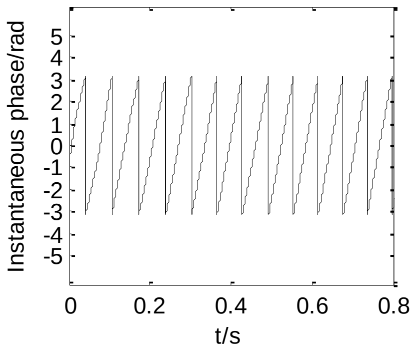


FIGURE 14 Phase in time of IMF2

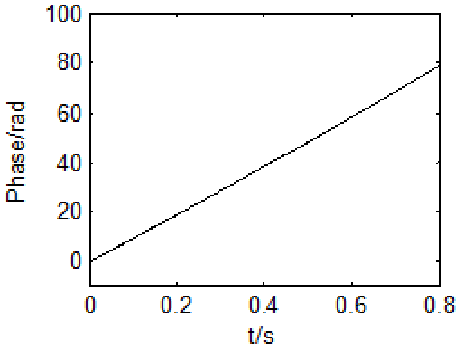


FIGURE 15 Continuous phase in time of IMF2

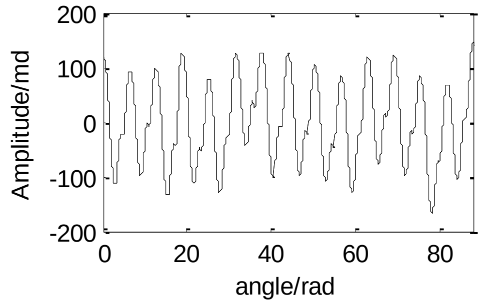


FIGURE 16 Torsional vibration signal in angle domain

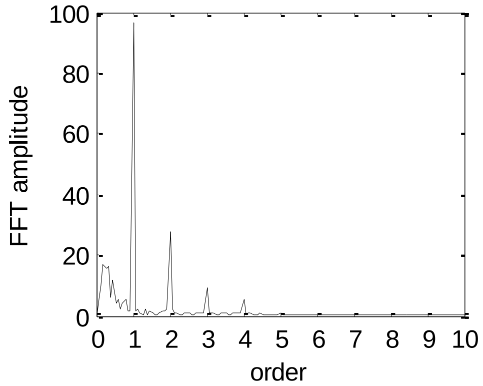


FIGURE 17 Order spectrum

The Signal order spectrum showed that the original torsinal vibration signal was mainly composed four harmonic components, and the amplitudes of each harmonic component were 96.1867md, 27.5165md, 9.005md and 5.5002md respectively. The continuous phase was processed by quadratic polynomial fitting, acquiring the phase function of time as shown in Equation (13). The angular acceleration obtained by the Equation

(13) was 6.2084/2=3.1043 rad/s^2, which was coincided with the actual angular acceleration. The result validates the reliability and accuracy of the order analysis method based on instantaneous phase.

6 Conclusions

The vibration signals of rotating machinery are usually compound signals relevant to the rotating frequency. Using HHT method, the instantaneous phases of the rotating frequency component and multiple frequency components can be extracted, which can be computed to the rotating angle, thus the time sequence of the even angle resample can be gained.

The non-stationary torsional vibration signals of rotating machinery were theoretically and experimentally analysed, using order analysis method based on instantaneous method. The order spectrums of the torsional vibration signals were achieved. The simulating and experimental results both verified the feasibility and accuracy of the proposed method. The proposed order analysis method takes advantage of the instantaneous phase obtained by HHT. Compared with the method using instantaneous frequency, this method is more convenient and faster, and it is less effected by the 'boundary effect', thus can obtain higher accuracy.

Acknowledgments

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