Chaos control of unified chaotic system base on tridiagonal matrix stability theory and adaptive hybrid synchronization

Yaolin Huang*, **Shuangli Wang, Chengjun Xie, Zhuo Jiang, Jinyan Zhao**

College of Computer Science and Technology, Beihua University, Jilin, 132021, China

Received 13 June 2014, www.cmnt.lv

Abstract

This paper presents a new chaos control method to control the unified chaotic system to zero. In order to control a unified chaotic system to zero, the first step is to design a stable system, which stable to zero base on tridiagonal matrix stability theory, the stable system as the master system. The second step is to make the unified chaotic system with controlled as slave system, the third step to make the master system and the slave system synchronization. Different system state apply different adaptive synchronize method to realize synchronization. The adaptive control law and parameter update law are obtained base on Lyapunov stability theory. Numerical simulations are presented to demonstrate the effectiveness of theoretical analysis.

Keywords: chaos control, adaptive hybrid synchronization, tridiagonal structure matrix stability theory

1 Introduction

In 1990, Ott, Grebogi and Yorke presented the OGY method on chaos control [1]. After their pioneering work, chaos control has gained more interest in nonlinear problems and there have been a lot of progress in this field [2-5]. Generally, there are two control ways: feedback control and nonfeedback control. Feedback methods [6-10] are used to stabilize the unstable periodic orbit of chaotic systems by feedback their states. Nonfeedback methods [11, 12] are adopted to suppress chaotic behaviour by apply periodic perturbations to some parameters or variables. With the development of chaos control technology, all kinds of methods are developed.

This paper proposes a new chaos control method to control the unified chaotic system to zero, the new method base on tridiagonal structure matrix stability and hybrid synchronization.

This paper is organized as follows. In the next section, we analyse the tridiagonal structure matrix stability theory and system with tridiagonal structure matrix. In section 3, we introduce the hybrid synchronization theory. In section 4, we make use of tridiagonal matrix stability theory and adaptive hybrid synchronization to control the unified chaotic system to zero. In section 5, some numerical simulations are done to test the effectiveness of theoretical analysis. Finally, some conclusions are drawn in section 6.

2 The theory

2.1 THE TRIDIAGONAL STRUCTURE MATRIX STABILITY THEORY

Lemma 1 [13, 14] If the nonlinear system has the following forms of tridiagonal structure:

$$\dot{X} = \begin{bmatrix} -k_1 & f_1(x) & & & \\ -f_1(x) & -k_2 & \ddots & & \\ & \ddots & \ddots & f_{n-1}(x) \\ & & -f_{n-1}(x) & -k_n \end{bmatrix} X \text{ , where}$$

$X(t) = (x_1, x_2, \cdots, x_n)^T \in R^n$ is state vectors, $f_j(x_i)(j = 1, 2, \ldots, n-1, i = 1, 2, 3, \ldots, n)$ are function about x_i, $k_i \in R^+(i = 1, 2, \ldots, n)$, the state vectors X of nonlinear system will be globally asymptotically stabilized to origin.

Proof: A positive definite function is as follows, $V = \frac{1}{2} X^T X$.

The derivative of V, $\dot{V} = -\sum_{i=1}^{n} K_i x_i^2 < 0.$

Due to V is the Lyapunov function and x will be globally asymptotically stabilized to origin [14].

2.2 SYSTEM WITH TRIDIAGONAL STRUCTURE MATRIX

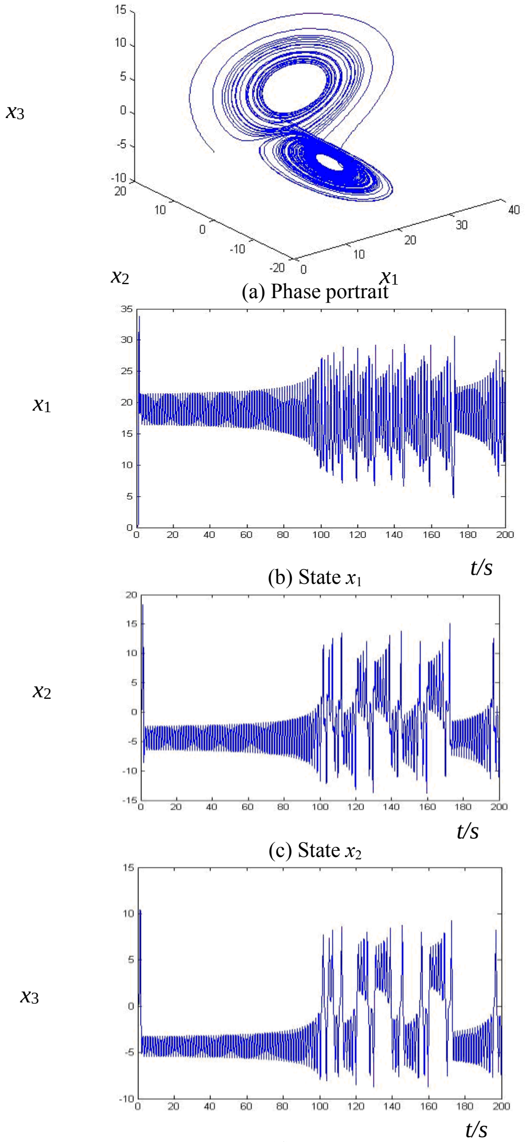
Due to permanent magnet synchronous motor (PMSM) system has tridiagonal structure, we introduce PMSM system.

* ***Corresponding author*** e-mail hyl200199@163.com

Model of PMSM can be expressed as follows [15],

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & x_3 & 0 \\ -x_3 & -1 & \gamma \\ 0 & \sigma & -\sigma \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}. \tag{1}$$

The parameters γ and σ ($\sigma > 0$) can decide the system (1) is chaos or not [16]. Fig. 1 show the chaotic character of system (1) with $\gamma = 20$ and $\sigma = 3$, initial values $x_1(0) = 0.1, x_2(0) = 0.2, x_3(0) = 0.4$. From Figuge 1 it can be concluded that system (1) is in chaotic state.



(a) Phase portrait

(b) State x_1

(c) State x_2

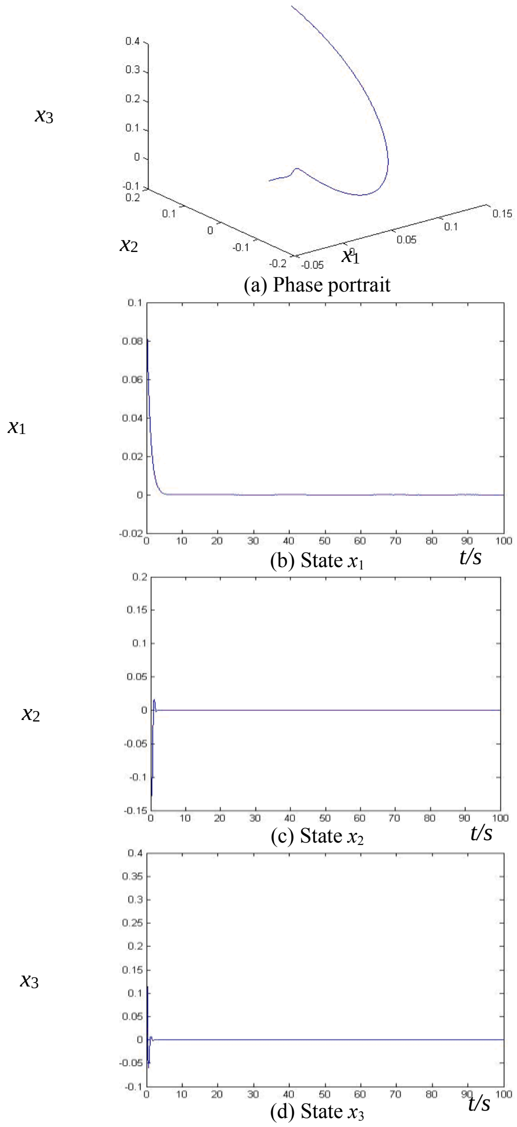
(d) State x_3

FIGURE 1 the chaotic character of system (1) when $\gamma = 20$ and $\sigma = 3$

System (1) has tridiagonal structure matrix form when parameters $\gamma = -\sigma$, which can be expressed as follows,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & x_3 & 0 \\ -x_3 & -1 & -\sigma \\ 0 & \sigma & -\sigma \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}. \tag{2}$$

Figure 2 shows states changes of system (2) when parameters $\sigma = 5$, initial values $x_1(0) = 0.1, x_2(0) = 0.2, x_3(0) = 0.4$. From Figure 2, it can be concluded that system (2) is stabilized to zero.



(a) Phase portrait

(b) State x_1

(c) State x_2

(d) State x_3

FIGURE 2 Trajectories of the system (2) states when $\sigma = 5$

3 Hybrid Synchronization

Consider the following three-dimensional master system (3) and slave system (4),

$$\dot{x} = f(t, x), \tag{3}$$

$$\dot{y} = g(t, y) + u(t, x, y), \tag{4}$$

where $x(t) = (x_1, x_2, x_3)^T \in R^3$ and $y(t) = (y_1, y_2, y_3)^T \in R^3$ are master and slave state vectors respectively, $f : R^3 \to R^3$ and $g : R^3 \to R^3$ are continuous nonlinear vector functions and $u(t, x, y) = (u_1, u_2, u_3)^T \in R^3$ is control input for synchronization between master system (3) and slave system (4).

Definition 1. It is said that hybrid synchronization occurs between master system (3) and slave system (4) if satisfy the following conditions:

Define $s(r)$ is a function, where r is state vector. Satisfy

$$\lim_{t\to\infty}\left\|e_1(t)=y_1(t)-s(x_1(t))\right\|=0.$$

Define λ is constant and satisfy

$$\lim_{t\to\infty}\left\|e_2(t)=y_2(t)-\lambda x_2(t)\right\|=0.$$

Define $m(t)$ is a function and satisfy

$$\lim_{t\to\infty}\left\|e_3(t)=y_3(t)-m(t)x_3(t)\right\|=0.$$

4 Method

The unified chaotic system [17] is expressed as follows,

$$\begin{aligned}\dot{x}_1&=(25\theta+10)(x_2-x_1)\\ \dot{x}_2&=(28-35\theta)x_1+(29\theta-1)x_2-x_1x_3\\ \dot{x}_3&=x_1x_2-\frac{8+\theta}{3}x_3\end{aligned}, \tag{5}$$

where $\theta\in[0,1]$. System (5) is chaotic for $\theta\in[0,1]$.

When $\theta\in[0,0.8)$, system (5) reduces to the general Lorenz system; when $\theta=0.8$, it becomes the general Lü system; and $\theta\in(0.8,1]$, system (5) is the general Chen system.

We assume that the system (2) with $\sigma=5$ as master system, system (5) with controlled as the slave system (6) given by,

$$\begin{aligned}\dot{y}_1&=(25\theta+10)(y_2-y_1)+u_1\\ \dot{y}_2&=(28-35\theta)y_1+(29\theta-1)y_2-y_1y_3+u_2\\ \dot{y}_3&=y_1y_2-\frac{8+\theta}{3}y_3+u_3\end{aligned}, \tag{6}$$

where u_1,u_2,u_3 are the control input.

By the definition of hybrid synchronization, the error is given by

$$\begin{aligned}e_1&=y_1-s_1(x_1)\\ e_2&=y_2-\lambda x_2\\ e_3&=y_3-m(t)x_3\end{aligned}. \tag{7}$$

The error dynamics system is then given as

$$\begin{aligned}\dot{e}_1&=\dot{y}_1-\dot{s}_1(x_1)\dot{x}_1\\ \dot{e}_2&=\dot{y}_2-\lambda\dot{x}_2\\ \dot{e}_3&=\dot{y}_3-\dot{m}(t)x_3-m(t)\dot{x}_3\end{aligned}. \tag{8}$$

By substituting Eqs. (2) And (6) in the above equation, we obtain

$$\begin{aligned}\dot{e}_1&=(25\theta+10)(y_2-y_1)-\dot{s}_1(x_1)(-x_1+x_3x_2)+u_1\\ \dot{e}_2&=(28-35\theta)y_1+(29\theta-1)y_2-y_1y_3-\lambda(-x_2-x_3x_1-5x_3)+u_2\\ \dot{e}_3&=y_1y_2-\frac{8+\theta}{3}y_3-\dot{m}(t)x_3-m(t)\left[5(x_2-x_3)\right]+u_3\end{aligned}. \tag{9}$$

Here, our goal is to achieve hybrid synchronization between two systems with different initial conditions. For this end, the following control laws are designed:

$$\begin{aligned}u_1&=-(25\hat{\theta}+10)(y_2-y_1)+\dot{s}_1(x_1)(-x_1+x_2x_3)-k_1e_1\\ u_2&=-(28-35\hat{\theta})y_1-(29\hat{\theta}-1)y_2+y_1y_3+\lambda(-x_2-x_3x_1-5x_3)-k_2e_2\\ u_3&=-y_1y_2+\frac{8+\hat{\theta}}{3}y_3+\dot{m}(t)x_3+5m(t)(x_2-x_3)-k_3e_3\end{aligned}, \tag{10}$$

where $\tilde{\theta}=\theta-\hat{\theta}$, we define $\hat{\theta}$ as estimate of θ of system (6), $\tilde{\theta}$ is estimate error, $k_i>0(i=1,2,3)$.

Substituting Eqs. (10) In Eqs. (9), we obtain

$$\begin{aligned}\dot{e}_1&=25\tilde{\theta}(y_2-y_1)-k_1e_1\\ \dot{e}_2&=-35\tilde{\theta}y_1+29\tilde{\theta}y_2-k_2e_2\\ \dot{e}_3&=-\frac{\tilde{\theta}}{3}y_3-k_3e_3\end{aligned}. \tag{11}$$

The parameter update rule for $\hat{\theta}$ is chosen as follows

$$\dot{\hat{\theta}}=\frac{1}{3}e_3y_3-25e_1(y_2-y_1)+e_2(35y_1-29y_2). \tag{12}$$

Theorem 1. The system (6) can be stabilized to zero by hybrid synchronization between master system (2) and slave system (6) when choose the control laws (10) and parameter update rule (12).

Proof. Choose the following Lyapunov function

$$V=\frac{1}{2}(e_1^2+e_2^2+e_3^2+\tilde{\theta}^2), \tag{13}$$

where $\tilde{\theta}=\theta-\hat{\theta}$, the time derivative of Eq. (13) along the trajectory of error system is

$$\dot{V}=\dot{e}_1e_1+\dot{e}_2e_2+\dot{e}_3e_3+\tilde{\theta}\dot{\hat{\theta}}, \tag{14}$$

and substitute Eqs. (11) and Eq. (12) in Eq. (14), $\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2$.

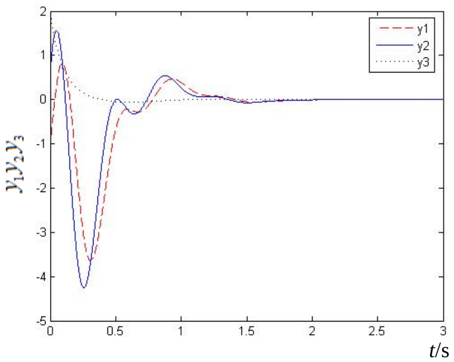
Since $\dot{V} < 0$, the master system (2) and the slave system (6) achieve synchronization. The slave system (6) can be stabilized to zero.

5 Numerical simulations

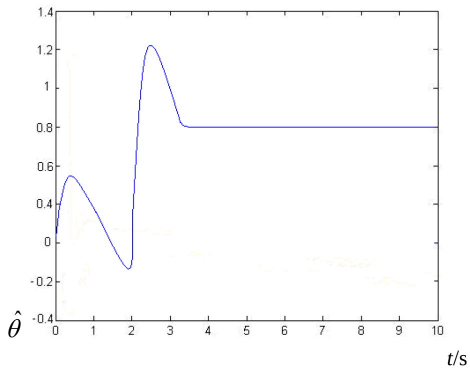
Numerical simulations are presented to demonstrate the effectiveness of the proposed synchronization controller. Fourth-order Runge-Kutta method is used to solve master system (2) and slave system (6) with time step size 0.0001, the parameter of master system (2) are chosen to be $\sigma = 5$ so that the master system (2) exhibits stable behaviour. The initial conditions of the master system (2) as follows, $x_1(0) = 0.1, x_2(0) = 0.2, x_3(0) = 0.3,$ and those of the slave system (6) are $y_1(0) = -1, y_2(0) = 0.6, y_3(0) = 2.$

The estimate parameter of slave system (6) is chosen as $\hat{\theta} = 0.1$ real parameter is $\theta = 0.8$. Moreover, $s_1(r) = \sin r, s_1(x_1) = \sin x_1$, $m(t) = \sin t + 1$, $\lambda = 30$. The control gains are chosen as $k_1 = 0.05$, $k_2 = k_3 = 10$.

The simulations results are illustrated in Fig. 3. From Fig. 3 (a) it can be concluded that system (6) can be stabilized to zero. From Fig. 3 (b) it can be concluded that and estimate parameter value research 0.8.



(a) Trajectories of the states y_1, y_2, y_3



(b) Parameter identification

FIGURE 3 System (6) control process

6 Conclusions

In this paper, we propose a chaos control method to control the unified chaotic system with uncertain parameter to zero. The first step is to design a stable system, which stabilizes to zero bases on tridiagonal matrix stability theory, the stable system as master system. The second step is to make the unified chaotic system with controlled as the slave system. The third step is to make the slave system and the master system synchronize. The variables of system apply adaptive function synchronization, adaptive projective synchronization and the adaptive generalized projective synchronization to realize the synchronization of master system and the slave system. Thus, the slave system can be stabilized to zero.

References

[1] Ott E, Grebogi C, Yorke J A 1990 Controlling chaos *Physical Review Letters* **64** 1196-9

[2] Arena A, Lacarbonara W 2012 Nonlinear parametric modeling of suspension bridges under aeroelastic forces: torsional divergence and flutter, nonlinear dynamics **70** 2487-510

[3] Hussain I, Shah T, Gondal M A 2012 A novel approach for designing substitution-boxes based on nonlinear chaotic algorithm, nonlinear dynamics **70** 1791-4

[4] Guan X P, Fan Z P, Chen C L, Hua C C 2002 *Chaotic control and its application on secure communication* Beijing: National Defence Industry Press 168-225

[5] Wang X Y 2003 *Chaos in the complex nonlinearity system* Beijing: Electronics Industry Press 28-32

[6] Jang M J, Chen C L, Chen C K 2002 Sliding mode control of hyperchaos in Rössler systems *Chaos, Solitons & Fractals* **14** 1465-76

[7] Wang X, Chen G, Yu X 2000 Anticontrol of chaos in continuous-time systems via time-delay feedback *Chaos* **10** 771-9

[8] Zheng Y A 2006 Controlling chaos using Takagi-Sugeno fuzzy model and adaptive adjustment *Chinese Physics* **15** 2549-52

[9] Gong L H 2005 Study of chaos control based on adaptive pulse perturbation *Acta Physica Sinaca* **54** 3502-7

[10] Wang X Y, Wu X J 2006 Chaos control of a modified coupled dynamos system *Acta Physica Sinaca* **55** 5083-93

[11] Wang L Z, Zhao W L 2005 Suppression of chaotic motion in a class of piecewise-smooth systems by using sine periodic force *Acta Physica Sinaca* **54** 4038-43

[12] Chen L, Wang D S 2007 Nonfeedback control of chen's chaotic system *Acta Physica Sinaca* **56** 91-4

[13] Liu B, Zhang Z 2007 Stability of nonlinear systems with tridiagonal structure and its applications *Acta Atomatica Sinica* **33** 442-5

[14] Liu B, Zhou Y M, Jiang M, Zhang Z K 2009 Synchronizing chaotic systems using control based on tridiagonal structure *Chaos Solitons & Fractals* **39** 2274-81

[15] Elmas C, Ustun O 2008 A hybrid controller for the speed control of a permanent magnet synchronous motor drive *Control Engineering Practice* **16** 260-70

[16] Wang X Y, Wang M J 2008 A hyperchaos generated from Lorenz system *Physica A* **387** 3751-8

[17] Lü J H, Chen R G, Cheng D Z 2002 Bridge the gap between the Lorenz system and the Chen system *Int J Bifur Chaos* **12** 2917

[18] Perchaotic Lü 2006 Attractor via state feedback control *Physica A* **364** 103-10

[19] Mohammad H, Mahsa D Impulsive synchronization of Chen

Authors

Huang Yaolin, born in July, 1979, Jilin, Jilin, China

Current position, grades: lecturer in the school of computer science and technology Beihua University, he and as school of computer science and technology college secretary, he was involved in the project of Jilin Provincial Department of science and technology, this project and won the three prize of scientific and technological progress in Jilin province.

Wang Shuangli, born in December, 1972, Jilin, Jilin, China

Current position, grades: associate professor at the school of computer science and technology, was a master's degree in engineering.
Publications: 2 papers in EI retrieval, the provincial papers 2, complete the student innovation project of national level 2. Is mainly engaged in object oriented programming, VC++ programming technology, network technology and Web technology in teaching and research activities, neural network and chaos theory, related project development and maintenance work practice.
Experience: In 2012 obtained the Jilin province science and technology progress prize 3 item, independently of the city Department of project 1, involved in the preparation of teaching materials 1

Xie Chengjun, born in September, 1963, Jilin, Jilin, China

Current position, grades: professor in the school of computer science and technology Beihua University, Ph.D. graduate, received a doctor degree in engineering. School of computer science and technology Beihua University Dean, top-notch innovative talents in Jilin Province, Jilin Province Department of education focus on training of young teachers, Jiangcheng elite talent, director of research Chinese computer education in higher normal school, director of the Jilin computer society, the expert review committee of project of national Natural Science Fund, hall of Jilin province science and technology project evaluation experts, master of computer application technology tutor, Beihua University computer professional academic leaders, academic committee of Beihua University, Journal of Beihua University (NATURAL SCIENCE EDITION) editorial board.

Jiang Zhuo, born in March, 1978, Jilin, Jilin, China

Current position, grades: lecturer in the school of computer science and technology, to obtain a master's degree in engineering, science and technology, she is also the director of computer
Scientific interest: mainly teaches data structure, software engineering, object oriented 8 courses analysis and design, guiding students curriculum design, graduation design. Presided over the completion of the "software engineering" quasi bilingual demonstration course project, participated in the "excellent course network teaching model research and practice", "local college computer professional basic course platform is arranged on the reform and practice of teaching research project"
Experience: participated in "software engineering" one textbook. Participate in program design, data structure, software engineering best construction. She also won the two prize of the Beihua University fourth session of young teachers and multimedia teaching contest, teaching excellence award. Beihua University outstanding teachers, outstanding communist title.

Zhao Jinyan, born in October, 1963, Jilin, Jilin, China

Current position, grades: professor at the school of computer science and technology, was a master's degree in engineering. She is also vice president Institute of computer science and technology.
Publications: 12 scientific research, editor of 2 books, published 9 papers
Experience: Once obtained the middle-aged backbone teacher, "outstanding communist", "Jilin city's plastic model", "three eight" red banner pacesetter in Jilin province and the educational system ethics advanced individual title. Access to the provincial teaching achievement 2 (first prize, three prize 1 items), the completion of the provincial research projects 3; awarded at provincial level or above education technology achievement award 5 items (including the Ministry of Education 1, Provincial Department of education, 4)